1. Waves Interacting with Themselves

1. Unit 6 on a Page
2. Introduction
3. Superposition and Interference
4. Optical Path Length
5. Why Optical Path Length Matters
6. Introduction to the Double Slit Experiment
7. Solving Through the Geometry of the Double Slit Problem

Unit 6 on a Page

- Amplitudes (NOT INTENSITIES) add
- If peak-to-peak or trough-to-trough you get complete constructive interference and the amplitude is

- If peak-to-trough then you get complete destructive interference and amplitude is zero
- Can measure distances with wavelengths: Optical path length

—

- Remember from Unit 2,

  changes in a material
- The difference in optical path length

  can tell us interference (motivated in your reading)
- If

  is an integer: Complete constructive
- If

  is a half-integer: Complete destructive
- Fast bouncing off slow: Add a

  —

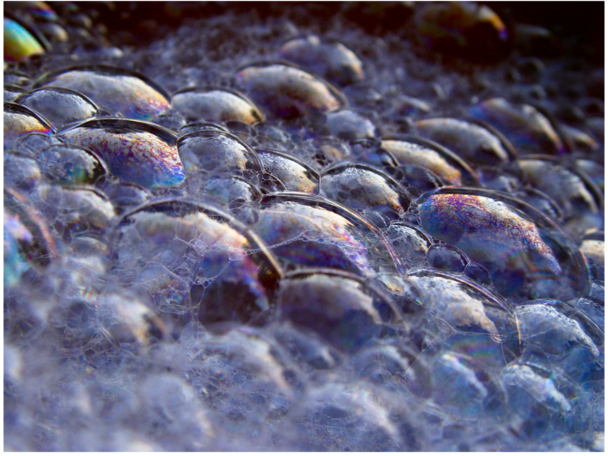
  to

Introduction

The colors reflected by this compact disc vary with angle and are not caused by pigments. Colors such as these are direct evidence of the wave character of light. (credit: Infopro, Wikimedia Commons)

Examine a compact disc under white light, noting the colors observed and locations of the colors. Determine if the spectra are formed by diffraction from circular lines centered at the middle of the disc and, if so, what is their spacing. If not, determine the type of spacing. Also with the CD, explore the spectra of a few light sources, such as a candle flame, incandescent bulb, halogen light, and fluorescent light. Knowing the spacing of the rows of pits in the compact disc, estimate the maximum spacing that will allow the given number of megabytes of information to be stored.

If you have ever looked at the reds, blues, and greens in a sunlit soap bubble and wondered how straw-colored soapy water could produce them, you have hit upon one of the many phenomena that can only be explained by the wave character of light (see [link]). The same is true for the colors seen in an oil slick or in the light reflected from a compact disc. These and other interesting phenomena, such as the dispersion of white light into a rainbow of colors when passed through a narrow slit, cannot be explained fully by geometric optics. In these cases, light interacts with small objects and exhibits its wave characteristics. The branch of optics that considers the behavior of light when it exhibits wave characteristics (particularly when it interacts with small objects) is called wave optics (sometimes called physical optics). It is the topic of this chapter.



These soap bubbles exhibit brilliant colors when exposed to sunlight. How are the colors produced if they are not pigments in the soap? (credit: Scott Robinson, Flickr)

Superposition and Interference

- Explain standing waves.
- Describe the mathematical representation of overtones and beat frequency.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

The key point of this section is that when two waves add, the resulting wave is a point-by-point addition of the two waves. You will need to do such additions and know the terms constructive and destructive.

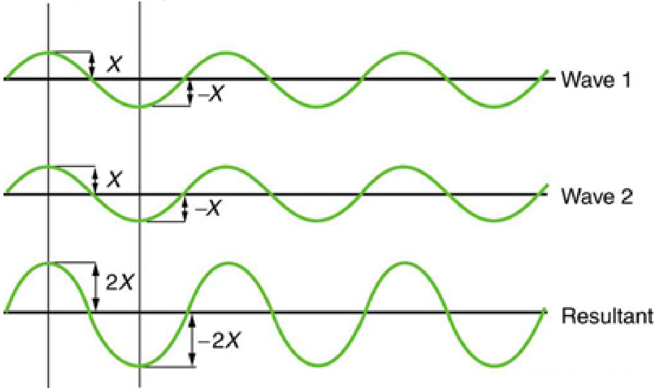
These waves result from the superposition of several waves from different sources, producing a complex pattern. (credit: waterborough, Wikimedia Commons)

Most waves do not look very simple. They look more like the waves in [link] than like the simple water wave considered in Waves. (Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape). Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

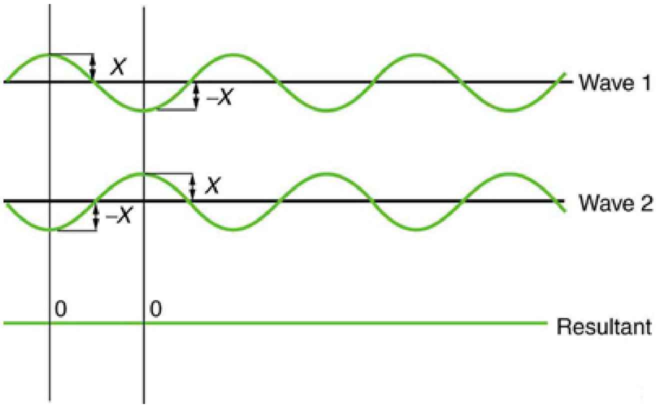
When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called **superposition**. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves—that is, their amplitudes add. [link] and [link] illustrate superposition in two special cases, both of which produce simple results.

[link] shows two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure **constructive interference**. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

[link] shows two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure **destructive interference**. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.



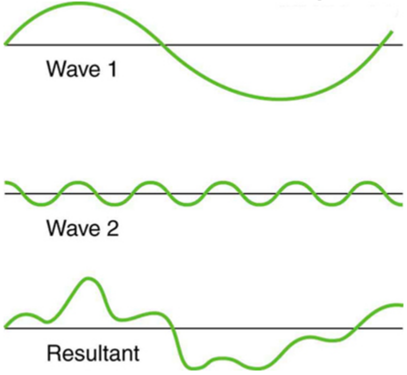
Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.



Pure destructive interference of two identical waves produces zero amplitude, or complete cancellation.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot and quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive. These examples are of waves that are similar.

An example of the superposition of two dissimilar waves is shown in [link]. Here again, the disturbances add and subtract, producing a more complicated looking wave.



Superposition of non-identical waves exhibits both constructive and destructive interference.

Exercise:
Check Your Understanding

Problem:

Imagine you are holding one end of a jump rope, and your friend holds the other. If your friend holds her end still, you can move your end up and down, creating a transverse wave. If your friend then begins to move her end up and down, generating a wave in the opposite direction, what resultant wave forms would you expect to see in the jump rope?

Solution:

The rope would alternate between having waves with amplitudes two times the original amplitude and reaching equilibrium with no amplitude at all. The wavelengths will result in both constructive and destructive interference

Exercise:

Check Your Understanding

Problem:

You hook up a stereo system. When you test the system, you notice that in one corner of the room, the sounds seem dull. In another area, the sounds seem excessively loud. Describe how the sound moving about the room could result in these effects.

Solution:

With multiple speakers putting out sounds into the room, and these sounds bouncing off walls, there is bound to be some wave interference. In the dull areas, the interference is probably mostly destructive. In the louder areas, the interference is probably mostly constructive.

Note:
PhET Explorations: Wave Interference
Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.

Wave Interference

Section Summary

- Superposition is the combination of two waves at the same location.
- Constructive interference occurs when two identical waves are superimposed in phase.

- Destructive interference occurs when two identical waves are superimposed exactly out of phase.

Glossary

antinode
the location of maximum amplitude in standing waves

beat frequency
the frequency of the amplitude fluctuations of a wave

constructive interference
when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

destructive interference
when two identical waves arrive at the same point exactly out of phase; that is, precisely aligned crest to trough

fundamental frequency
the lowest frequency of a periodic waveform

nodes
the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

overtones
multiples of the fundamental frequency of a sound

superposition
the phenomenon that occurs when two or more waves arrive at the same point

Optical Path Length
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as a video on the UMass Physics 13X YouTube page. The link can be found here.
By the end of this section you should:

- Know what an optical path length is
- Be able to calculate the optical path length in a variety of situations, both using graphical counting methods and situations with different materials.

To understand what optical path length is we have to go back to what may seem to be a very fundamental question: when I say that something is

$$10$$

meters long, what do I actually mean?

Below we have a figure of a Tarbosaurus, which is about

$$10$$

meters long. What do we mean by that? We mean that a Tarbosaurus is as long as 10-meter sticks end to end. We conventionally use sticks one meter long to measure things, but that's not the only thing we could use, we could use sticks one inch long or one foot long or we could say how many people long, or we could measure how long this Tarbosaurus is in wavelengths of a particular color of light.

For example, let's say we had a radio wave with a wavelength of

$$2$$

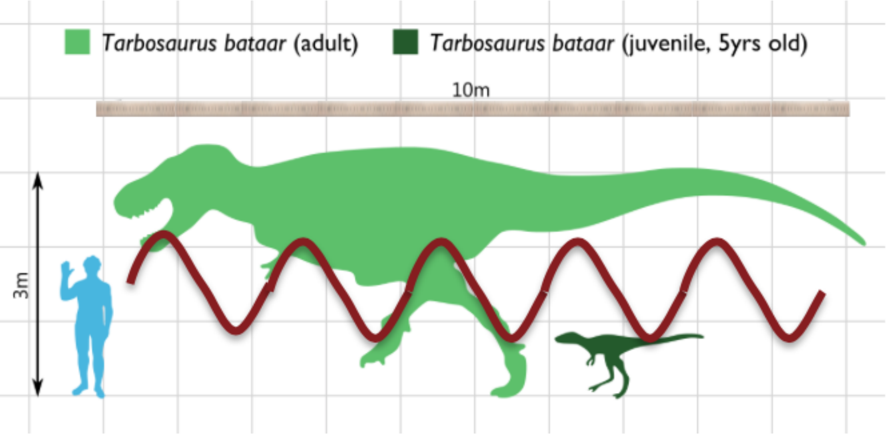
meters, then the Tarbosaurus would be

$$5$$

wavelengths long, which means it's

$$5$$

wavelengths of 2-meter light.


🟩 *Tarbosaurus bataar* (adult)
🟩 *Tarbosaurus bataar* (juvenile, 5yrs old)
10m
3m

But there's no reason we have to use a 2-meter radio wave, we could use a 600-nanometer light wave of the typical red laser pointer. How many wavelengths long is a Tarbosaurus in this unit? Well

$$10$$

meters, and we know that one wavelength is

$$680 \times 10^{-9}$$

meters, which gives us

$$1.471 \times 10^{7}$$

wavelengths of

$$680$$

nanometer light.

This is the fundamental principal of optical path length, you're measuring distances using wavelengths of light instead of meter sticks.

In the formula below for optical path length, we will use

$$l$$

is the distance traveled

$$x$$

divided by the wavelength.
Equation:

$$l = \frac{x}{\lambda}$$

What are the units of optical path length? Well optical path length is a distance, that travelled by the light, divided by another distance, the wave length, since both

$$x$$

and

$$\lambda$$

are distances and measured in meters, optical path length will be unitless. Optical path length is the number of wavelengths traveled by the light. We’re just counting wavelengths and things that are counted should be unitless. When I say that I have five apples, that's just five there's no unit on it.

There is one quirk with using wavelengths to count as a measure of distance and that comes in with materials. As you know from Unit 2 on geometric optics, the wavelength of light changes when it enters a material. The speed of light goes from the speed,

$$c$$

,

$$3\times10^8$$

, down to velocity,

$$v$$

, whatever the

$$v$$

is in the material, and we can find the index of refraction,

$$n$$

.

Equation:

$$n = \frac{c}{v}$$

When we combine this definition of

$$n$$

with the fundamental relationship for waves,

$$v = \lambda f$$

, and recognizing that the frequency can't change, this tells us that the wavelength must change to the wavelength in the material,

$$\lambda$$

must be

$$\lambda_0$$

, which is the wavelength in vacuum divided by the index of refraction,

$$n$$

.

Equation:

$$\lambda = \frac{\lambda_0}{n}$$

The key point for optical path length is that you should always use the wavelength of the material that the light is currently in.

Here's an example with some materials. Imagine that we have a source which emits light of

$$500$$

nanometer wavelength. The source is

$$3$$

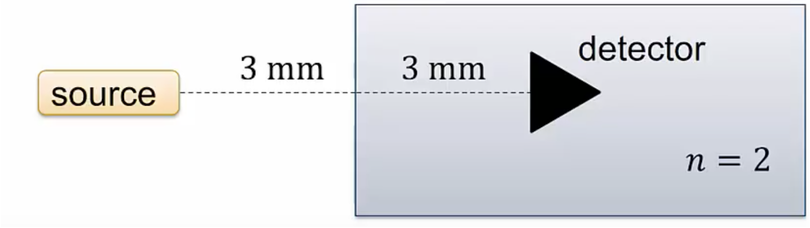
millimeters from a glass slab with an index of refraction

$$n = 2$$

. Buried within the slab another

$$3$$

millimeters inside is a detector. What is the optical path length between the source and the detector?



Well in this situation, we have two different regions each with a different lambda outside the glass and inside the glass. We have to do this problem in two parts.

First let's do the part outside the glass. The optical path length before the glass is the distance over the wavelength we know the distance is

$$3$$

millimeters, and we know the wavelength is

$$500$$

nanometers, which when you divide that out gives us an optical path length before the glass of

$$6{,}000$$

.

Now let's do the second part inside the glass. The optical path length inside the glass is the distance the light travels divided by the wavelength of the light in the glass. So, to get the wavelength in the glass, we take the wavelength of light in vacuum,

$$500$$

nanometers, and divide it by

$$2$$

to get

$$250$$

nanometers.

Now we can proceed to calculate the optical path length.

$$3$$

millimeters,

$$3\times10^{-3}$$

meters, divided by the wavelength of light in the glass,

$$250\times10^{-9}$$

meters will give us an optical path length in the glass of

$$12{,}000$$

. To get the total optical path length we then just add these two values, so the total optical path length is

$$6{,}000$$

plus

$$12{,}000$$

which is

$$18{,}000$$

.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

In Summary:

- Optical path length is essentially measuring by counting how many wavelengths something is long instead of how many meter sticks, which we will represent mathematically as

  $$l$$

  , for the optical path length, which is going to be equal to the distance traveled,

  $$x$$

  , divided by the wavelength

$$\lambda$$

,

$$l = \frac{x}{\lambda}$$

.

- Since different kinds of light have different wavelengths the optical path length even for the same distance can vary depending upon the wavelength of light being used, think back to our example with the dinosaur, it was

  $$5$$

  radio waves or

  $$1.47{\times}10^7$$

  $$680$$

  mm nanometer light.
- It's also worth remembering that the optical path length is a unitless quantity and finally you need to remember that you need to use the wavelength that is true where you happen to be, so you need to consider any indices of fraction.

Why Optical Path Length Matters

Exercise:

UMASS AMHERST Instructor's Notes

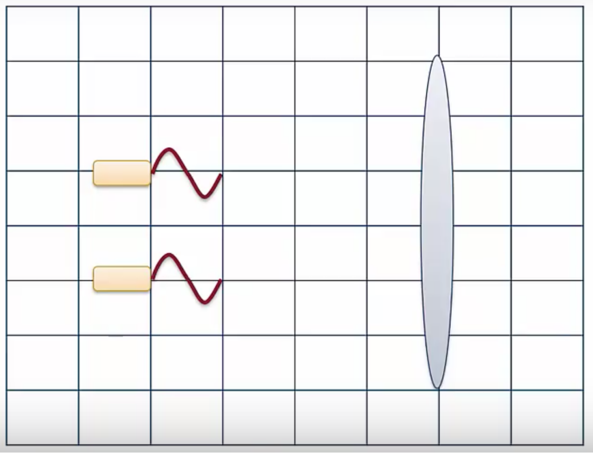
Problem:

This section is also available as a video on the [UMass Physics 13X YouTube page.](#) The link can be found [here.](#)

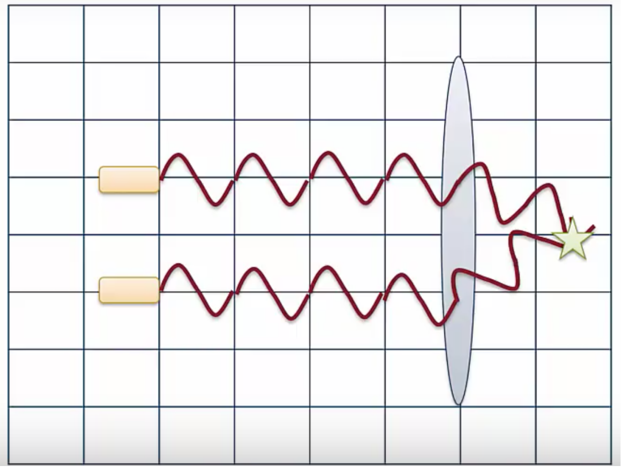
This section will really demonstrate the key fundamental principle for all of Unit 6 for this particular course.

Let's begin with an example.

I've laid a grid on the space so that we can see what's going on and let's add two light sources, each of which is emitting light coherently, so coming out of the sources the lights peak together and drops together, and then let's add a lens to bring these two incoming parallel light waves to a focal point as we discussed in Unit 2.

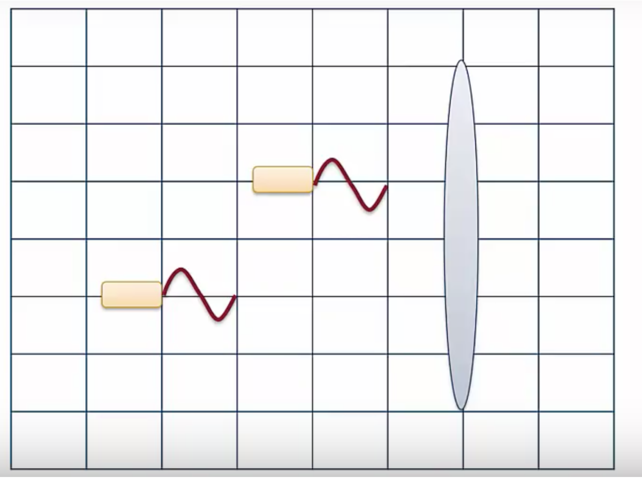


Now let's let the light travel so each light ray has as shown here travelled a few wavelengths. You can see at this point just before the sixth wavelength completes, the two waves, where they meet, have lined up trough to trough. Which means we're going to have constructive interference at that point.

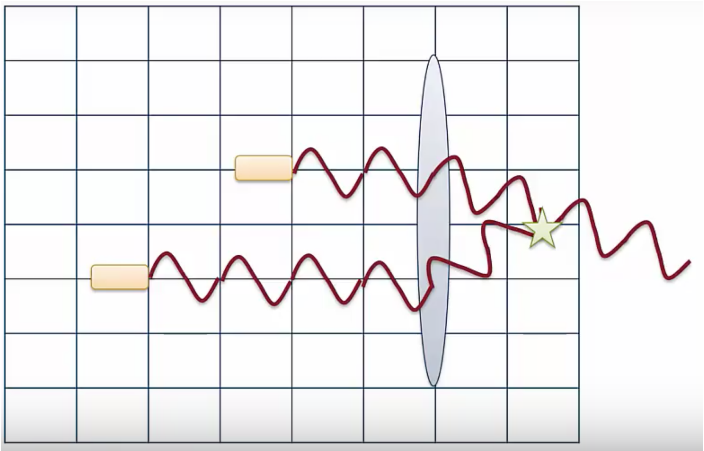


In general, anytime two light waves travel the same optical path length, they're going to travel the same number of wavelengths, they're always going to line up either peak to peak or trough to trough. When the optical path lengths are the same you're going to have constructive interference.

Now let's repeat the experiment but shift the top source of light to the right two wavelengths.



After a few wavelengths, the bottom wave hits the lens and is bent. You can see again the waves line up the trough to trough.



To get to that particular point, the top wave has traveled

$$3\frac{3}{4}$$

wavelengths, while the bottom wave has traveled

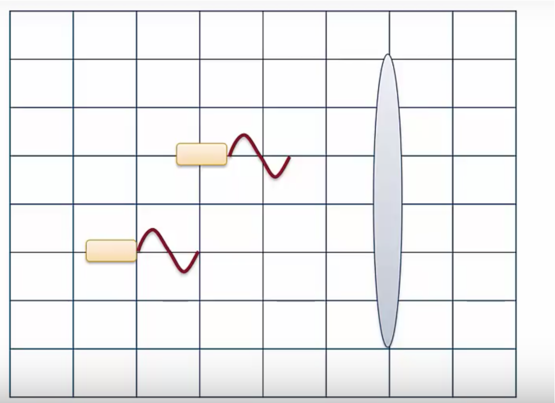
$$5\frac{3}{4}$$

. So, the difference in optical path length is 2 wavelengths, and we still get constructive interference.

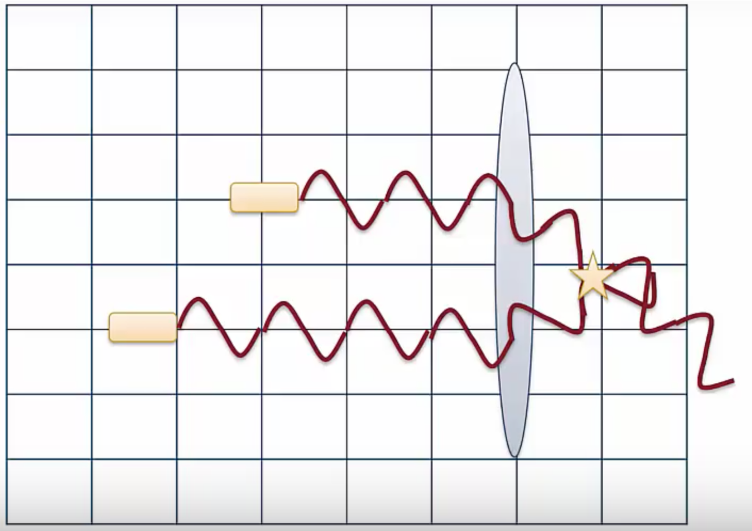
Now let's repeat the experiment for a third time, but this time only shift the top source to the right

$$1\frac{1}{2}$$

wavelengths.



Let's let the light travel a few wavelengths. The top wave has begun to bend, and now you can see at this point the top wave is on a trough and the bottom wave is on a peak so they're going to be destructively interfering.



At this point the top wave has traveled

$$3\frac{3}{4}$$

wavelengths, and the bottom wave has traveled

$$5\frac{1}{4}$$

wavelengths. The difference in optical path length here is therefore

$$1\frac{1}{2}$$

wavelengths, and so we have destructive interference.

Let's generalize these results. Anytime the difference in optical path length is an integer, so

$$1$$

,

$$2$$

,

$$-2$$

,

$$-3$$

,

$$0$$

, we get complete constructive interference, the peaks line up with the peaks or the troughs line up with the troughs. Meanwhile when the difference in optical path length is a half integer,

$$\frac{1}{2}$$

,

$$\frac{3}{2}$$

,

$$\frac{-5}{2}$$

etc., we get complete destructive interference, a peak will line up with a trough a trough a line up with a peak. Mathematically, we will call this difference in optical path length

$$\delta$$

, and it's just the difference in the two path lengths. So,

$$\delta = 11 - 12$$

.

Exercise:

UMASS AMHERST Instructor's Notes

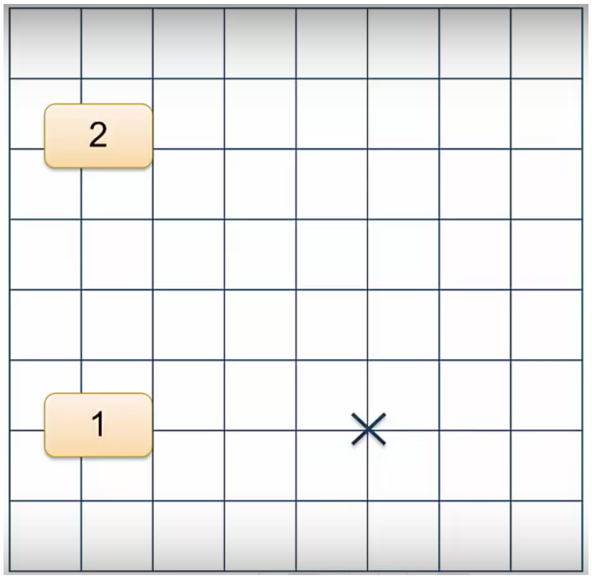
Problem:

This is the big idea of all of Unit 6. All we're going to be doing for the rest of the unit in class is comparing optical path lengths to see if the difference in optical path length is an integer or half integer for a variety of different geometries in situations. But I want to stress, this is essentially all we are going to be doing for the entirety of this unit, so you should keep this in mind.

Let's do another example. So here we have our two sources still emitting light coherently, but this time let's look at this little point

$$x$$

here. We're going to solve this problem graphically first, and then we'll go mathematically.



We're going to use a wavelength of

$$1$$

square. The wave from Source 1 travels

$$3$$

wavelengths to get to

$$x$$

. The wave from Source 2 on the other hand requires

$$5$$

wavelengths to get to

$$x$$

and we can see that the difference,

$$3-5=-2$$

, and at the

$$x$$

the wavelengths line up peak-to-peak, and so the Sources add up constructively.

Now let's solve this same problem mathematically. The wavelength is again

$$1$$

, the distance for Source 1 is

$$3$$

,

$$x$$

is equal to

$$3$$

. The wavelength is

$$1$$

so the optical path length is

$$\frac{3}{1} = 3$$

. For Source 2 on the other hand, to get the distance

$$x$$

we can use the Pythagorean theorem, because we can see we have a triangle here with one leg of

$$3$$

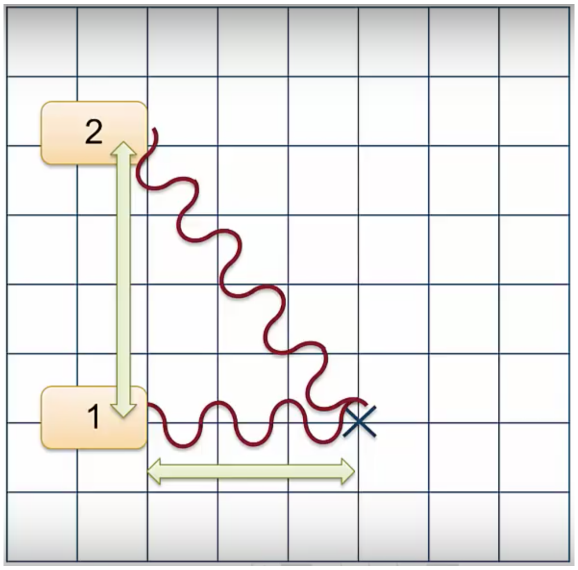
and the other leg of

$$4$$

,

$$3^2 + 4^2 = 25$$

.



This means the distance traveled by the light is

$$\sqrt{25}$$

or

$$5$$

units. Since the wavelength is still

$$1$$

, the optical path length will be

$$5$$

. The difference in optical path length,

$$l_1 - l_2$$

, will be

$$3 - 5 = -2$$

. Which we know to be an integer and so that tells us that we're going to get constructive interference.

Here's another example without a picture this time. Let's say we have some sort of system with two light rays each with a wavelength of

$$700$$

nanometers. One ray travels

$$5250$$

nanometers the other travels

$$9100$$

nanometers, what happens when the two rays are brought together?

I want to stress, it doesn't matter which light ray I call Source 1 and which light ray I call Source 2, flipping the order and calling the

$$5250nm$$

Source 2 will just change the sign. We don't care what the value of the difference in optical path length is, we just care is if it's an integer or a half integer so I'm free to call whichever one I want Source 1 and whichever one I want Source 2.

I'm going to call the light ray that travels

$$5250$$

nanometers Source 1 for this particular light ray. The distance is

$$5250$$

nanometers the wavelength is

$$700$$

nanometers, which gives us an optical path length of

$$\frac{5250}{700} = 7\frac{1}{2}$$

. The first ray travels

$$7\frac{1}{2}$$

wavelengths to get from the source to the point where the two rays are brought together.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

As long as the units for

$$x$$

and

$$\lambda$$

match I can use whatever units I want. This is one of the few cases where I don't have to convert everything to meters first as long as the distances are the same, our goal is to get the optical path length to be unitless.

If the

$$5250$$

is Source 1 that makes the other Source 2, so what's the optical path length for this light ray?

Well the distance,

$$9100$$

nanometers, divided by wavelength,

$$700$$

nanometer, gives us

$$\frac{9100}{700} = 13$$

. Which means the difference in optical path length will be

$$\delta = 7.5 - 13$$

and so we can tell that at the point where the two rays are brought together we will have destructive interference.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

I want to stress again this is all we will be doing in this entire unit this particular calculation with a variety of different configurations of light sources and mirrors and so forth.

In Summary:

- the key quantity for the study of interference is the difference in optical path length

  $$\delta = l_1 - l_2$$

  .
- If this Delta is an integer you can know that you will have complete constructive interference, if this Delta is a half integer you know that you will have complete destructive interference.

- We don't care what value Delta takes, and we don't care what the sign of Delta is, all that matters is if it is an integer or half integer.

Introduction to the Double Slit Experiment

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as a video on the UMass Physics 13X YouTube page. The link can be found here.

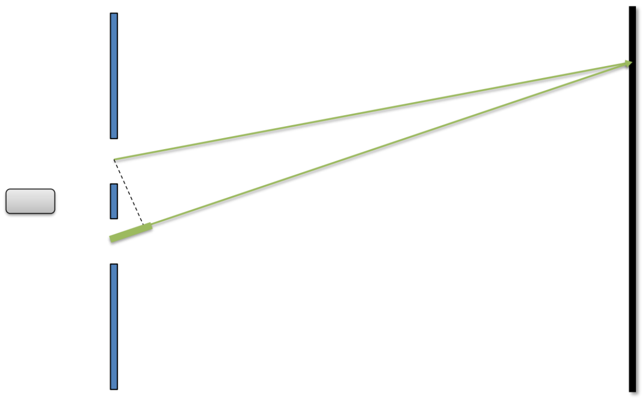
Now we're going to go through the double-slit, we've been talking about this experiment since like day two. Now we're going to talk about why it works. But it's still the same principle that we're talking about, the difference in optical path lengths. You've been seeing this in lab as a laser, pair of slits, and a screen.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

You've been seeing this in lab as a laser, pair of slits, and a screen.



Let's go and look at that path difference. We have some of the laser light goes through the top slit, and some of it goes thought the bottom slit. What's the difference? How much further does the bottom travel than the top? That's the question we're looking at, what's the extra distance?

Well if we look at it, the bottom one is going to have to go an extra little bit of distance to get to that point because it starts further away. You can see we're still already looking at those differences in path length. We are going to color the two rays red and purple to make it easier to talk, and the green is the extra distance.

Now here comes the geometry. The angle

$$\theta$$

is the angle from the center line to the origin. The angle from the center line to the point we're talking about is

$$\theta$$

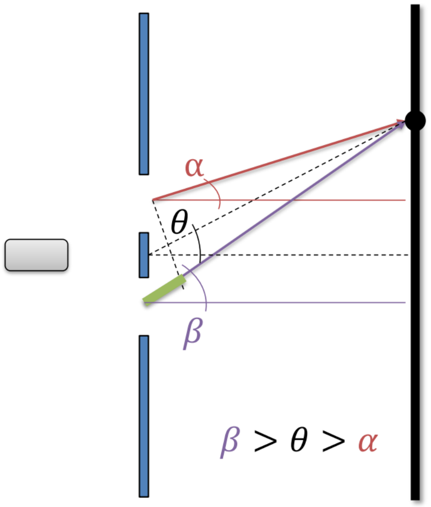
. There's also an angle for the top array,

$$\alpha$$

. There's also an angle for the bottom array which we shall call

$$\beta$$

.



If you have a look,

$$\beta$$

is a bigger angle than

$$\theta$$

, and

$$\theta$$

is a bigger angle than

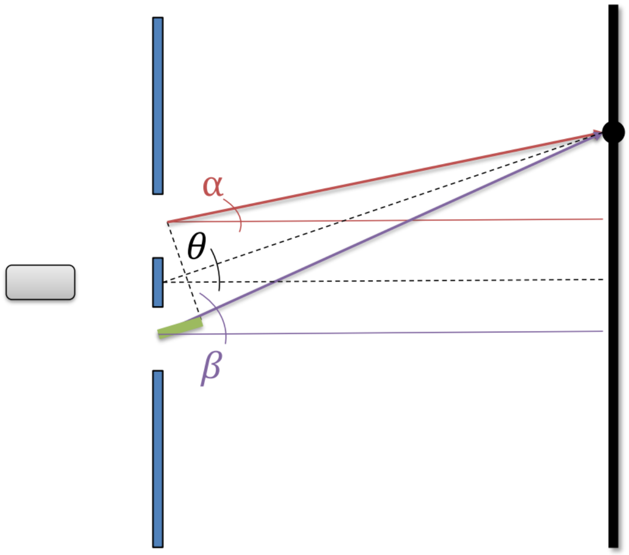
$$\alpha$$

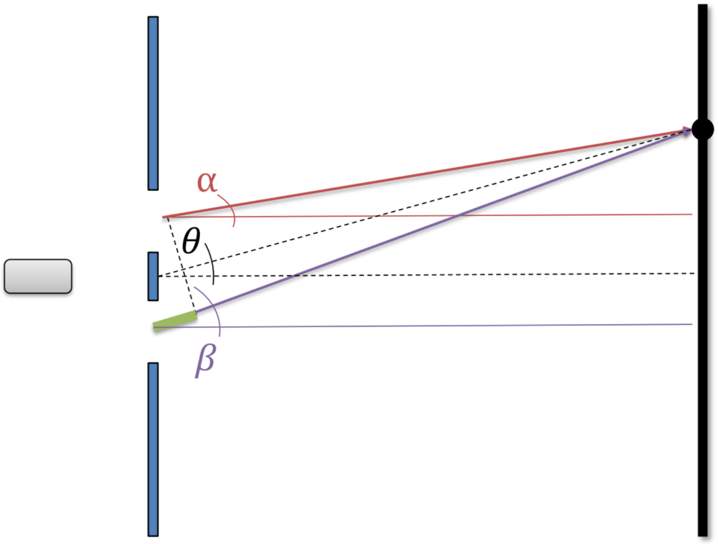
. These angles are different, and

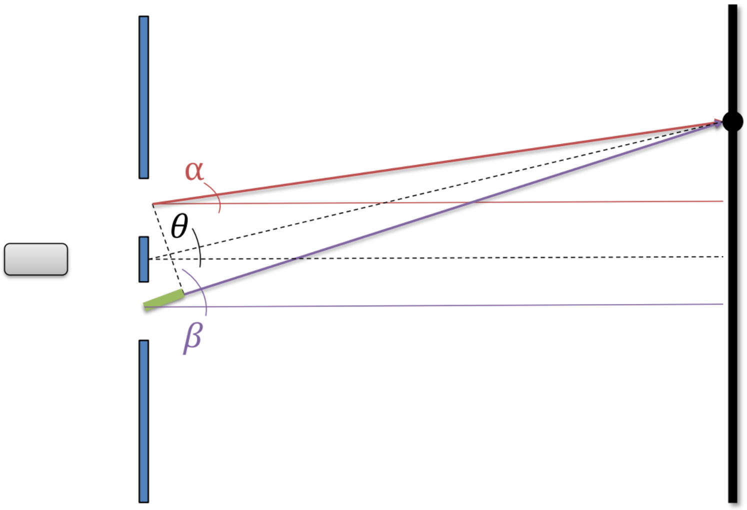
$$\beta$$

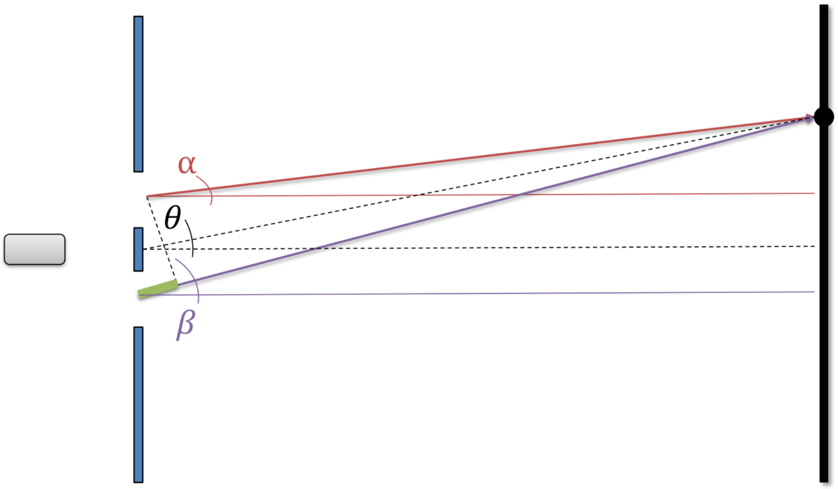
is the biggest

Now let's take this screen and move it move it back.









As we've moved it back, all the angles get smaller and closer together. They don't all get smaller at the same rate, but

$$\beta$$

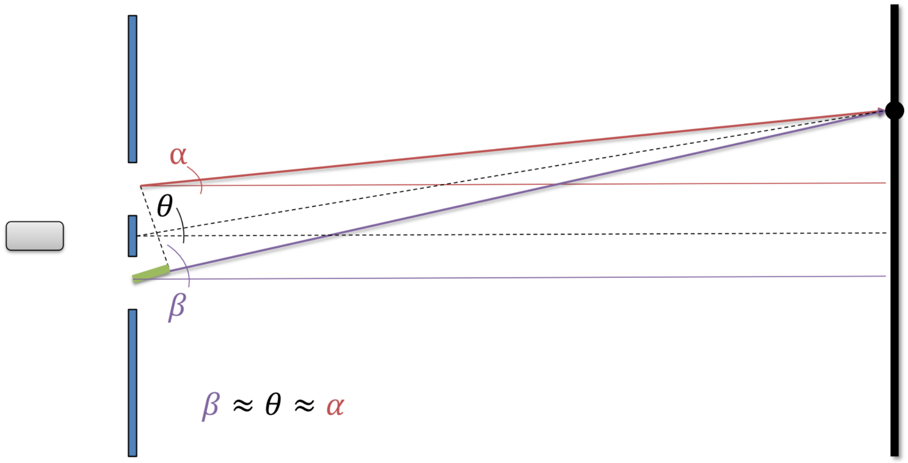
,

$$\alpha$$

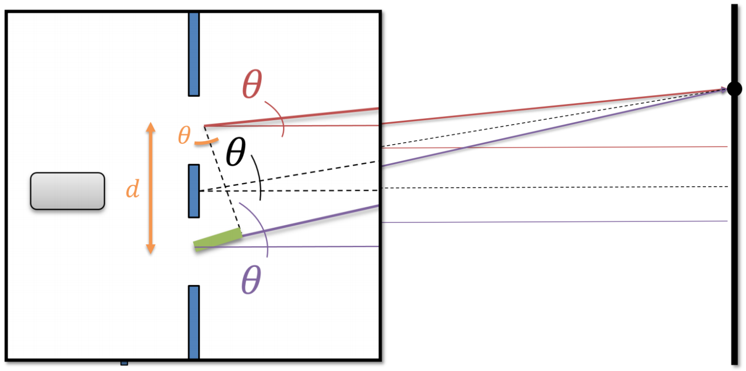
, and

$$\theta$$

all become very close to the same angle.



Now let's zoom in on just what's going on near the slits. Remember the green line is the extra distance that the purple ray has to travel.



That angle,

$$\theta,$$

turns out to be the same as all of these other angles. If the distance between the two slits is

$$d$$

, the extra distance over

$$d$$

is going to be

$$sin\theta$$

, so the extra distance is going to be

$$d \ \ sin\theta$$

,

$$\delta$$

is always extra distance over

$$\lambda$$

. And so that gives us the

$$\delta$$

for the double-slit.

Equation:

$$\delta \qquad \frac{d \qquad \theta}{\lambda}$$

Exercise:

UMASS AMHERST Instructor's Notes

Problem: The equation above will be on your equation sheet.

Solving Through the Geometry of the Double Slit Problem

About What You Are Reading

What follows is an explanation of this geometry by a fellow student, Catherine Trewhella, in her own words. I have even included the figures exactly as she drew them, simply copying them to white paper in ink for visual clarity.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

Trying to write such textbook sections is a great way to enhance your understanding of a piece of material about which you are confused, and I would encourage you to try this technique as you study!

The Problem

A double slit has

$$5\mu m$$

between the slits and is placed

$$20m$$

from the screen. If I am looking at a point

$$11.54^{\circ}$$

above the center-line, what is the difference in optical path length if

$$\lambda - 500nm$$

?

Solving it: Getting Started

Convert the numbers given into the correct units & draw out a picture of a basic double slit problem: Distance between slits

$$d$$

:
Equation:

$$d = (5\mu m)\bullet\left(\frac{1\mu m}{1\times10^{-6}m}\right) = 5\times10^{-6}m$$

Distance from screen:

$$y = 20m$$

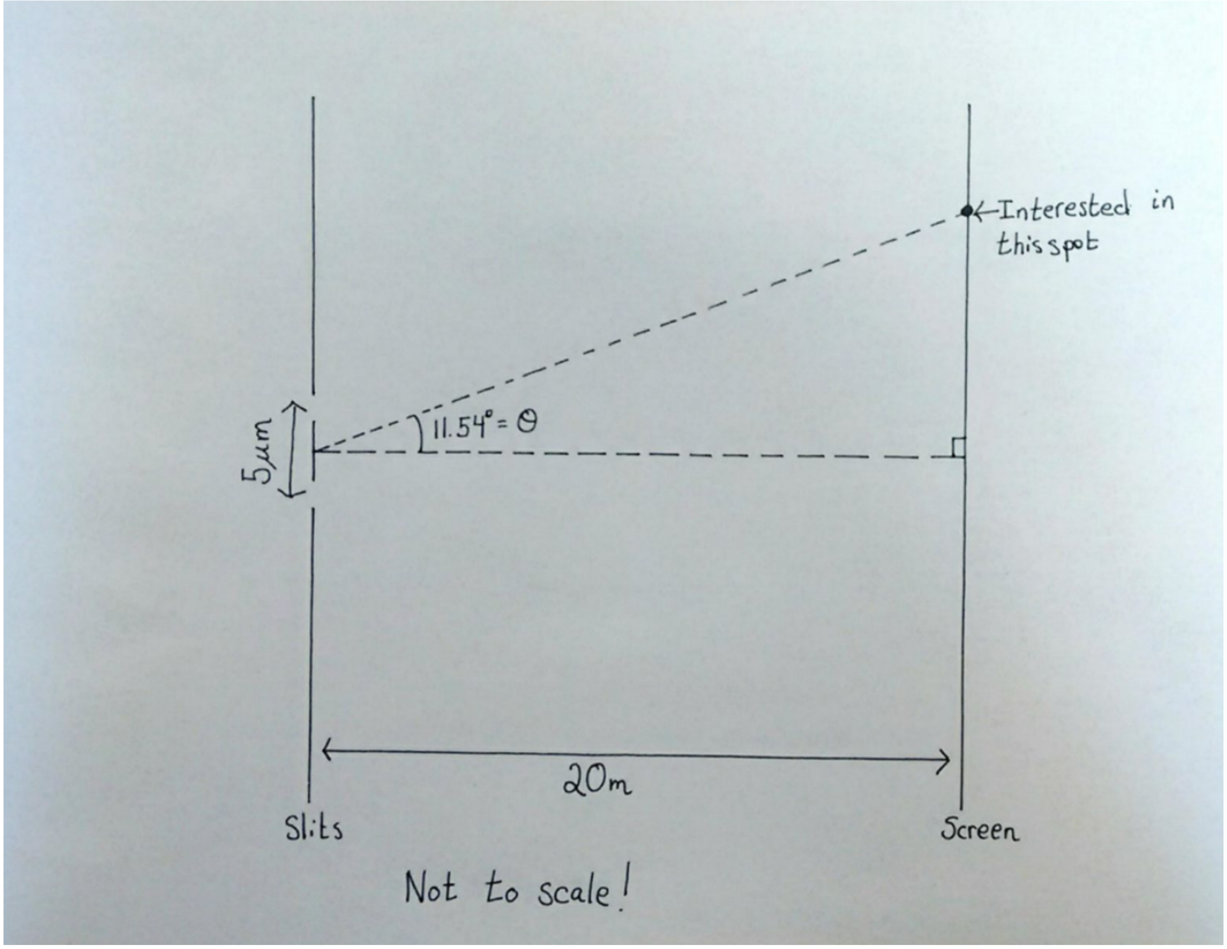
Wavelength

$$\lambda$$

:
Equation:

$$\lambda = (500\mu m)\bullet\left(\frac{1\mu m}{1\times10^{-9}m}\right) = 5\times10^{-7}m$$

We then sketch a figure of the setup, note it does not need to be to scale!



The Approach
The approach to solving this problem is breaking down the physics behind

$$\delta = l_1 - l_2$$

. By finding the distance, in meters, for

$$l_1$$

and

$$l_2$$

using geometry, we can convert them to wavelengths, and plug them into the equation.

Drawing

$$l_1$$

and

$$l_2$$

Draw in

$$l_1$$

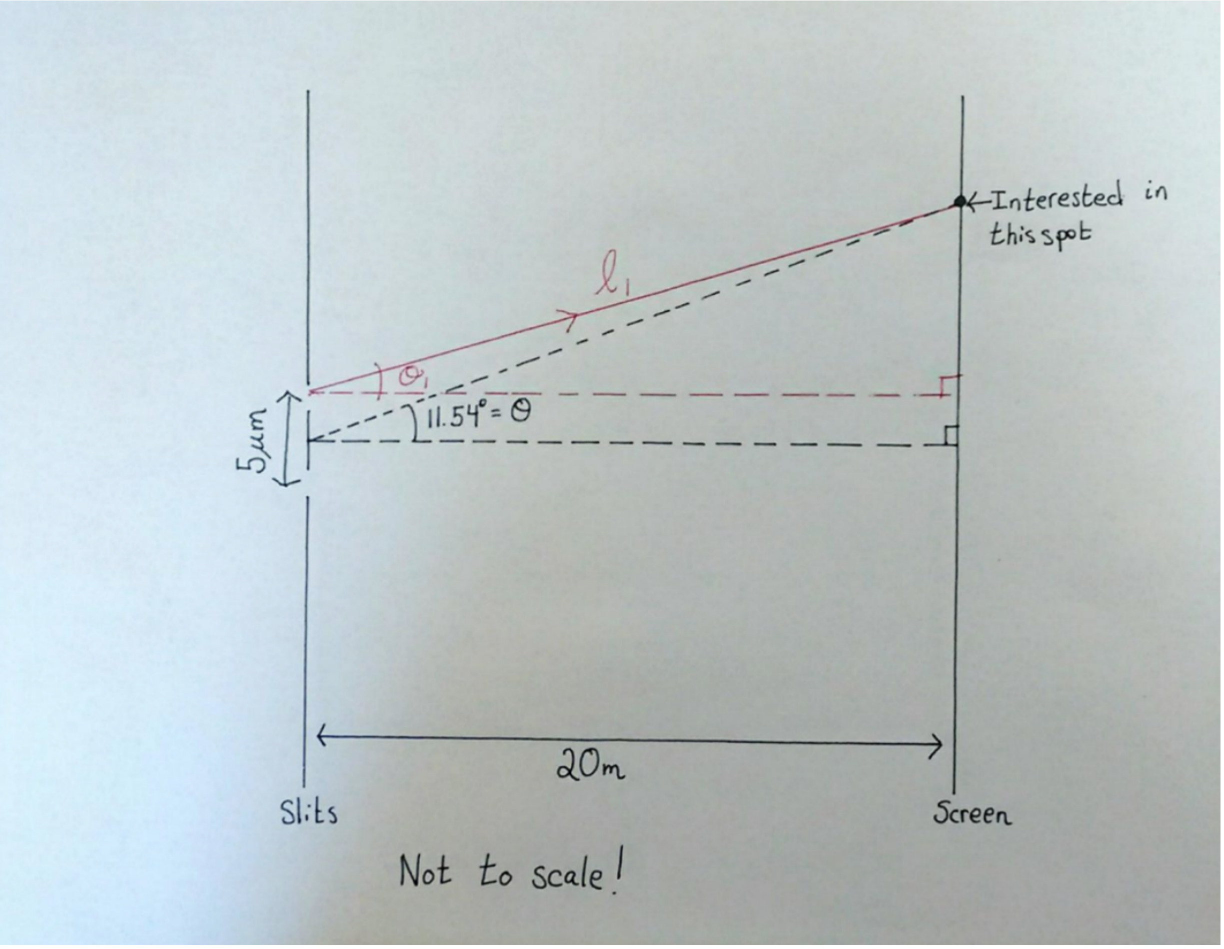
by adding a line in red that connects the upper slit to the screen, making a triangle. The angle does not need to be perfectly to scale! This represents the light travelling from the top slit to the point on our screen. The ray

$$l_1$$

makes an angle

$$\theta_1$$

.



Draw in

$$l_2$$

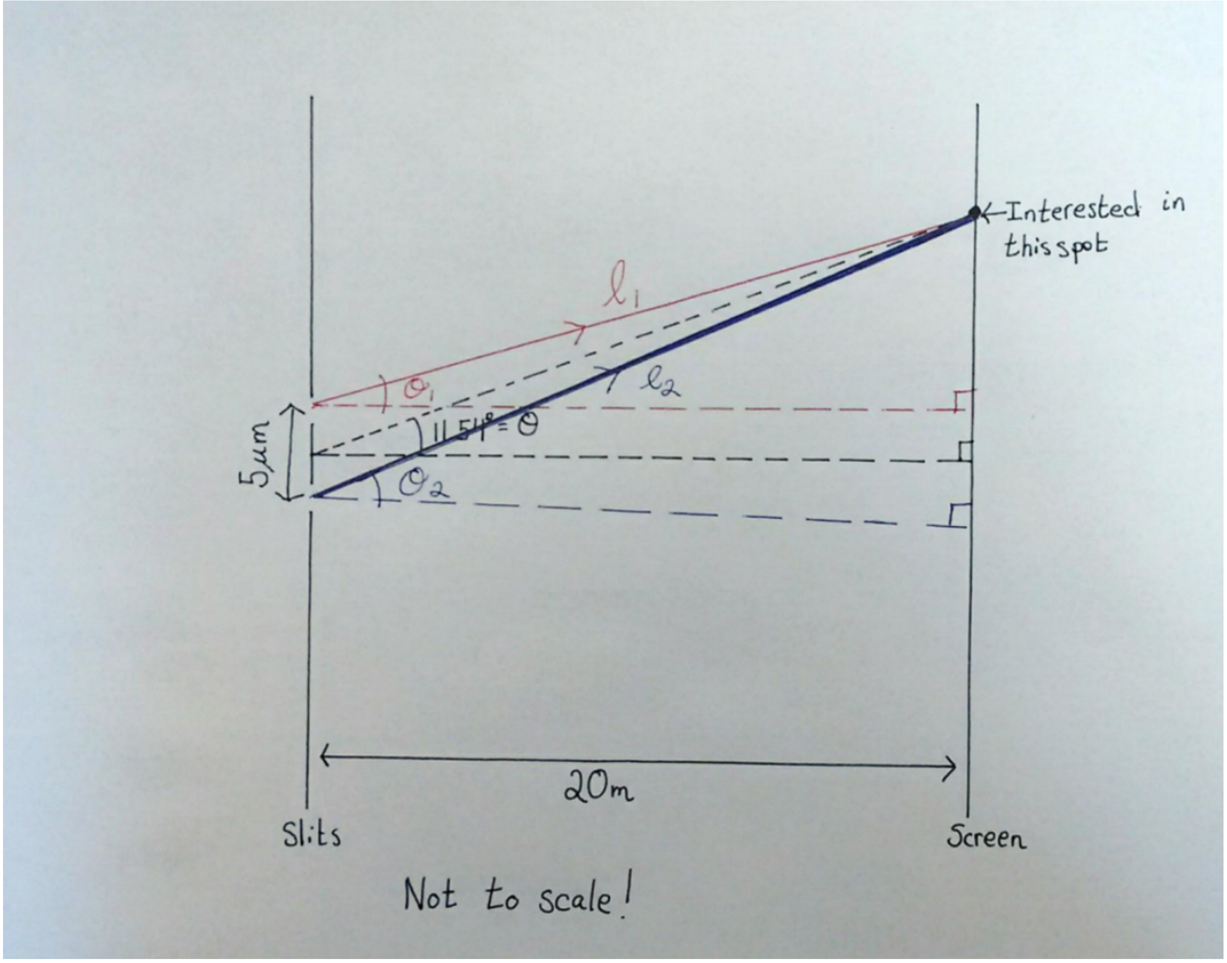
by adding a line in blue that connects the bottom slit to the same point on the screen, making a second triangle. The angle again does not need to be perfectly to scale! This represents the light travelling from the bottom slit to the point on our screen. The ray

$$l_2$$

makes an angle

$$\theta_2$$

.



The all of these angles (

$$\theta_1$$

,

$$\theta_2$$

, and

$$\theta = 11.54^\circ$$

) are essentially the same. On a micro scale, the differences in the angle matter, however the screen can be moved until the

$$\theta$$

’s become equivalent. It is similar to seeing leaves on a tree up close but moving farther back all of the leaves look the same. So, we take

$$\theta_1 \approx \theta_2 \approx \theta \approx 11.54^\circ$$

.

Solving for

$$l_2$$

(red triangle)
Since

$$l = \frac{x}{\lambda}$$

, we must next solve for the length of

$$x_1$$

, in meters using our understanding of geometry for right triangles, or as many remember it “SOH-CAH-TOA”. Since we are looking for the value of our hypotenuse, and only have the angle and the adjacent side, from the slit to the screen, we need to use the equation for cosine:
Equation:

$$cos\theta = \frac{adj}{hyp}$$

Equation:

$$hyp = \frac{adj}{cos\theta}$$

Equation:

$$x_1 = \frac{20m}{\cos\left(11.54^\circ\right)}$$

Equation:

$$x = 20.412415m$$

However, since this is

$$x_1$$

in meters and we want

$$l_1$$

, we must pull from previous units and convert the distance in meters to the number of wavelengths, similar to any other conversion.
Equation:

$$l_1 = \frac{x_1}{\lambda} = \frac{20.412415m}{5{\times}10^{-7}m} = 40824829$$

This

$$l_1$$

is unitless because it is the number of wavelengths that occur between the two points, not the actual distance.

Solving for

$$l_2$$

(blue triangle)
Since all angles are

$$11.54^{\circ}$$

more geometry must be used to solve for

$$l_2$$

by finding the difference between

$$x_1$$

and

$$x_2$$

in meters, and adding it to

$$x_1$$

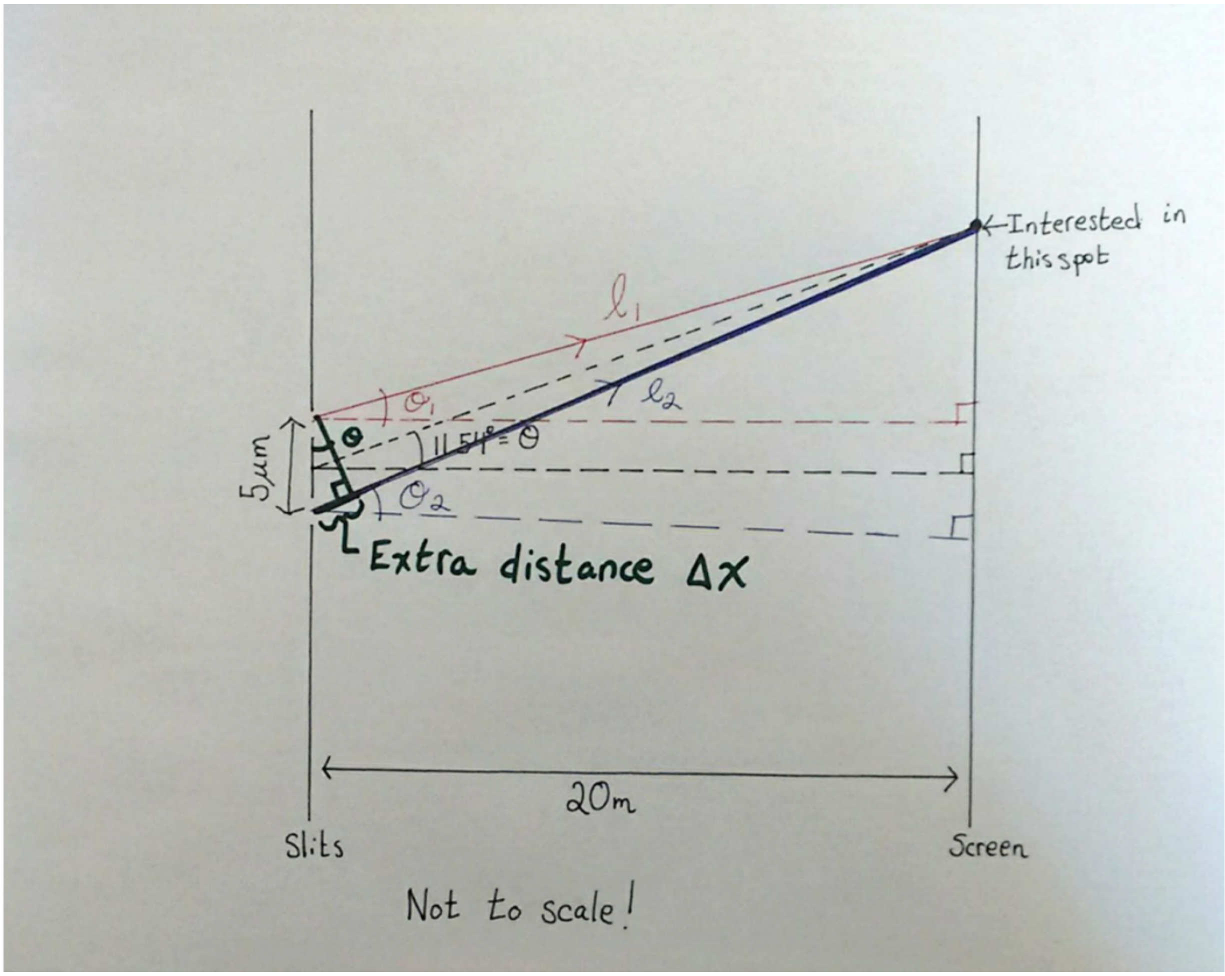
in meters before converting to the number of wavelengths. Start this process by drawing another line starting at the corner of the top slit and hitting at

$$90^\circ$$

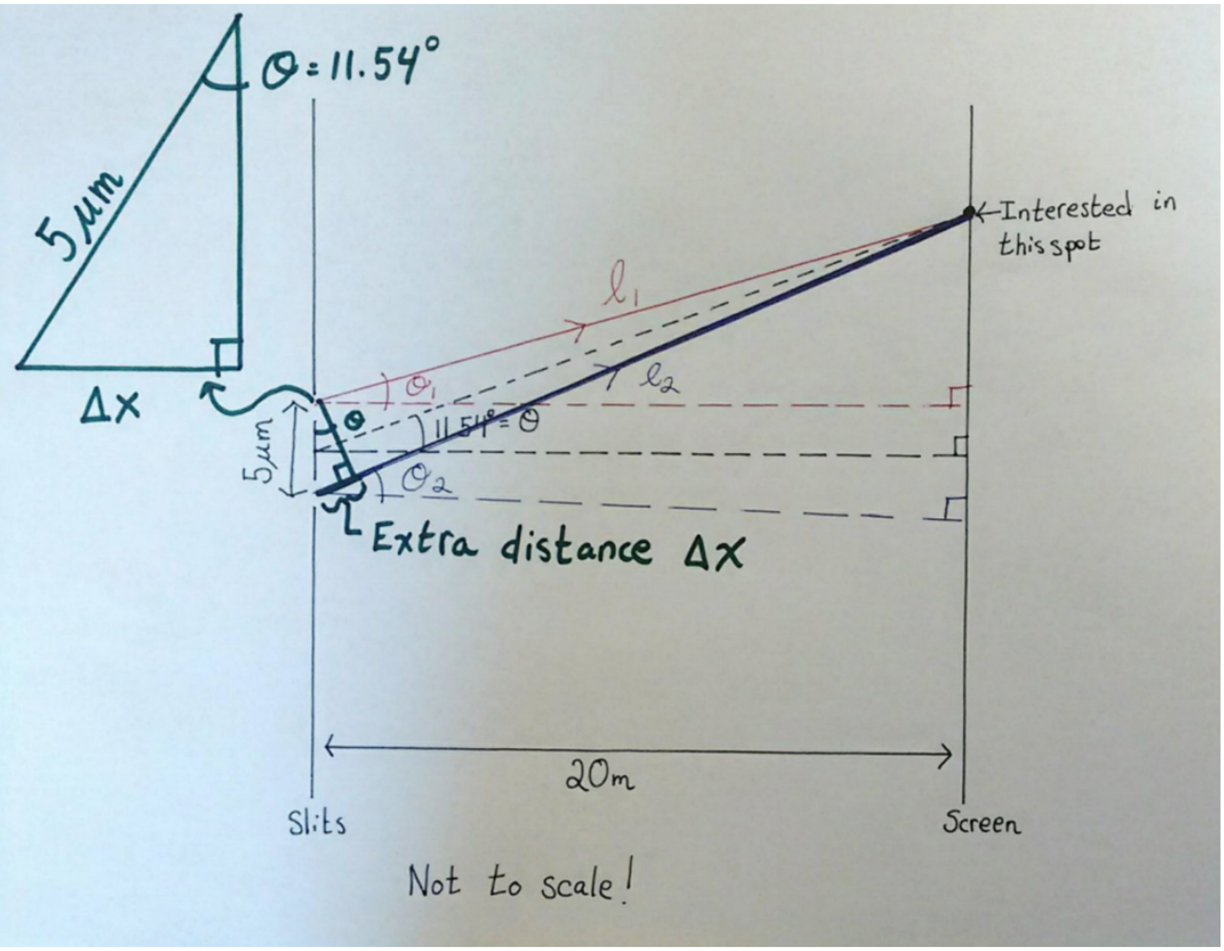
creating a third right triangle. The angle made is also equivalent to

$$\theta = 11.54^\circ$$

.



The new triangle can be turned on its side, to better show that it is a right triangle.



The difference in meters between

$$l_1$$

and

$$l_2$$

will be identified as

$$\Delta x$$

, and can be solved using the equation for sine, as we have values for the angle and hypotenuse.

Equation:

$$sin\theta = \frac{opp}{hyp}$$

Equation:

$$opp = hyp\bullet sin\theta$$

Equation:

$$x = (5\times10^{-6}m)\bullet\sin\left(11.54^{\circ}\right)$$

Equation:

$$\mathit{\Delta}x = 0.000001 = 1\times10^{-6}m$$

Now to solve for the actual distance

$$x_2$$

in meters, we must add this

$$\mathit{\Delta}x$$

value to

$$x_1$$

.

Equation:

$$x_2 = x_1 + \mathit{\Delta}x$$

Equation:

$$x_2 = (20.412415m) + (0.000001m)$$

Equation:

$$x_2 = 20.412416m$$

And now convert this

$$x_1$$

to the number of wavelengths to get

$$l_2$$

:

Equation:

$$l_2 = \frac{x_2}{\lambda} = \frac{20.412416m}{5{\times}10^{-7}m} = 40824831$$

Finally look at how these two rays will interfere

Now we have both the

$$l_1$$

and

$$l_2$$

values and can solve for

$$\delta$$

.

Equation:

$$\delta = l_1 - l_2$$

Equation:

$$\delta = 40824829 - 40824831$$

Equation:

$$\delta = -2$$

Meaning that this point will be a spot of constructive interference (a bright spot!).

Now that you understand the third triangle that was used to find

$$l_2$$

, you can skip the last section of steps we did, and just solve for the number of wavelengths within

$$\Delta x$$

to check for errors, as it should be the same as

$$l_1 - l_2$$

. This is the principle behind

$$x_{d.s.} = dsin\theta$$

on your equation sheet.

Conceptual Questions

Exercise:

Problem:

Speakers in stereo systems have two color-coded terminals to indicate how to hook up the wires. If the wires are reversed, the speaker moves in a direction opposite that of a properly connected speaker. Explain why it is important to have both speakers connected the same way.

title

Exercise:

Problem:

A car has two horns, one emitting a frequency of 199 Hz and the other emitting a frequency of 203 Hz. What beat frequency do they produce?

Solution:

$f = 4\text{ Hz}$

Exercise:

Problem:

The middle-C hammer of a piano hits two strings, producing beats of 1.50 Hz. One of the strings is tuned to 260.00 Hz. What frequencies could the other string have?

Exercise:

Problem:

Two tuning forks having frequencies of 460 and 464 Hz are struck simultaneously. What average frequency will you hear, and what will the beat frequency be?

Solution:

462 Hz,

4 Hz

Exercise:

Problem:

Twin jet engines on an airplane are producing an average sound frequency of 4100 Hz with a beat frequency of 0.500 Hz. What are their individual frequencies?

Exercise:

Problem:

A wave traveling on a Slinky® that is stretched to 4 m takes 2.4 s to travel the length of the Slinky and back again. (a) What is the speed of the wave? (b) Using the same Slinky stretched to the same length, a standing wave is created which consists of three antinodes and four nodes. At what frequency must the Slinky be oscillating?

Solution:

(a) 3.33 m/s

(b) 1.25 Hz

Exercise:

Problem:

Three adjacent keys on a piano (F, F-sharp, and G) are struck simultaneously, producing frequencies of 349, 370, and 392 Hz. What beat frequencies are produced by this discordant combination?